

Smooth Cosmologies from M-theory^{*}

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ABSTRACT

We review two ways in which smooth cosmological evolution between two de Sitter phases can be obtained from M/string-theory. Firstly, we perform a hyperbolic reduction of massive type IIA^{*} theory to $D = 6$ $\mathcal{N} = (1, 1)$ $SU(2) \times U(1)$ gauged de Sitter supergravity, which supports smooth cosmological evolution between $dS_4 \times S^2$ and a dS_6 -type geometry. Secondly, we obtain four-dimensional de Sitter gravity with $SU(2)$ Yang-Mills gauge fields from a hyperbolic reduction of standard eleven-dimensional supergravity. The four-dimensional theory supports smooth cosmological evolution between $dS_2 \times S^2$ and a dS_4 -type geometry. Although time-dependent, these solutions arise from a first-order system *via* a superpotential construction. For appropriate choices of charges, these solutions describe an expanding universe whose expansion rate is significantly larger in the past than in the future, as required for an inflationary model.

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1 $D = 6$ $\mathcal{N} = (1, 1)$ de Sitter supergravity from massive type IIA*

The advantage of embedding de Sitter solutions in $*$ theories [1, 2] is that they can be viewed as “supersymmetric,” even though the anti-commutators of the super-charges are no longer positive definite¹. The time-like T-dualization of type IIB theory can yield massive type IIA* supergravity, whose bosonic Lagrangian is given by [4]

$$\begin{aligned}\mathcal{L}_{10} = & \hat{R} \hat{*} \mathbf{1} - \frac{1}{2} \hat{*} d\hat{\phi} \wedge d\hat{\phi} + \frac{1}{2} e^{\frac{3}{2}\hat{\phi}} \hat{*} \hat{F}_{(2)} \wedge \hat{F}_{(2)} - \frac{1}{2} e^{-\hat{\phi}} \hat{*} \hat{F}_{(3)} \wedge \hat{F}_{(3)} \\ & + \frac{1}{2} e^{\frac{1}{2}\hat{\phi}} \hat{*} \hat{F}_{(4)} \wedge \hat{F}_{(4)} - \frac{1}{2} d\hat{A}_{(3)} \wedge d\hat{A}_{(3)} \wedge \hat{A}_{(2)} - \frac{1}{6} m d\hat{A}_{(3)} \wedge (\hat{A}_{(2)})^3 \\ & - \frac{1}{40} m^2 (\hat{A}_{(2)})^5 + \frac{1}{2} m^2 e^{\frac{5}{2}\hat{\phi}} \hat{*} \mathbf{1},\end{aligned}\tag{1}$$

where $\hat{F}_{(2)} = d\hat{A}_{(1)} + m \hat{A}_{(2)}$, $\hat{F}_{(3)} = d\hat{A}_{(2)}$ and $\hat{F}_{(4)} = d\hat{A}_{(3)} + \hat{A}_{(1)} \wedge d\hat{A}_{(2)} + \frac{1}{2} m \hat{A}_{(2)} \wedge \hat{A}_{(2)}$. The hyperbolic reduction² of massive type IIA* theory can be obtained as an analytical continuation of the S^4 reduction [8] of massive type IIA supergravity [9]:

$$\begin{aligned}ds_{10}^2 &= c^{\frac{1}{12}} X^{\frac{1}{8}} \left[\Delta^{\frac{3}{8}} ds_6^2 + 2g^{-2} \Delta^{\frac{3}{8}} X^2 d\xi^2 + \frac{1}{2} g^{-2} \Delta^{-\frac{5}{8}} X^{-1} s^2 \xi \sum_{i=1}^3 (h^i)^2 \right], \\ \hat{F}_{(4)} &= -\frac{\sqrt{2}}{6} g^{-3} c^{1/3} s^3 \Delta^{-2} U d\xi \wedge \epsilon_{(3)} - \sqrt{2} g^{-3} c^{4/3} s^4 \Delta^{-2} X^{-3} dX \wedge \epsilon_{(3)} \\ &\quad - \sqrt{2} g^{-1} c^{1/3} s X^4 *F_{(3)} \wedge d\xi - \frac{1}{\sqrt{2}} c^{4/3} X^{-2} *F_{(2)} \\ &\quad + \frac{1}{\sqrt{2}} g^{-2} c^{1/3} s F_{(2)}^i h^i \wedge d\xi \\ &\quad - \frac{1}{4\sqrt{2}} g^{-2} c^{4/3} s^2 \Delta^{-1} X^{-3} F_{(2)}^i \wedge h^j \wedge h^k \epsilon_{ijk}, \\ \hat{F}_{(3)} &= c^{2/3} F_{(3)} + g^{-1} c^{-1/3} s F_{(2)} \wedge d\xi, \\ \hat{F}_{(2)} &= \frac{1}{\sqrt{2}} c^{2/3} F_{(2)}, \quad e^{\hat{\phi}} = c^{-5/6} \Delta^{1/4} X^{-5/4},\end{aligned}\tag{2}$$

where $X = e^{-\frac{1}{2\sqrt{2}}\phi}$, $\Delta \equiv -X s^2 \xi + X^{-3} c^2 \xi$ and $U \equiv X^{-6} c^2 + 3X^2 s^2 - 4X^{-2} s^2 - 6X^{-2}$. We have defined $h^i \equiv \sigma^i - g A_{(1)}^i$, $\epsilon_{(3)} \equiv h^1 \wedge h^2 \wedge h^3$, $s = \sinh \xi$, $c = \cosh \xi$ and $m = \frac{\sqrt{2}}{3} g$. σ_i are $SU(2)$ left-invariant 1-forms which satisfy $d\sigma^i = -\frac{1}{2} \epsilon_{ijk} \sigma^j \wedge \sigma^k$. $*$

¹The first de Sitter solution within the context of an extended supergravity theory was found in [3].

²No-go theorems prohibiting de Sitter compactifications [5, 6] are surmounted by reducing over a non-compact space [7].

is the six-dimensional Hodge dual. The resulting six-dimensional theory³ is given by

$$\begin{aligned}
\mathcal{L}_6 = & R * \mathbb{1} - \frac{1}{2} * d\phi \wedge d\phi + g^2 \left(\frac{2}{9} e^{\frac{3}{\sqrt{2}}\phi} - \frac{8}{3} e^{\frac{1}{\sqrt{2}}\phi} - 2e^{-\frac{1}{\sqrt{2}}\phi} \right) * \mathbb{1} \\
& - \frac{1}{2} e^{-\sqrt{2}\phi} * F_{(3)} \wedge F_{(3)} + \frac{1}{2} e^{\frac{1}{\sqrt{2}}\phi} \left(* F_{(2)} \wedge F_{(2)} + * F_{(2)}^i \wedge F_{(2)}^i \right) \\
& + A_{(2)} \wedge \left(\frac{1}{2} dA_{(1)} \wedge dA_{(1)} + \frac{1}{3} g A_{(2)} \wedge dA_{(1)} \right. \\
& \left. + \frac{2}{27} g^2 A_{(2)} \wedge A_{(2)} + \frac{1}{2} F_{(2)}^i \wedge F_{(2)}^i \right),
\end{aligned} \tag{3}$$

where $F_{(3)} = dA_{(2)}$, $F_{(2)} = dA_{(1)} + \frac{2}{3} g A_{(2)}$ and $F_{(2)}^i = dA_{(1)}^i + \frac{1}{2} g \epsilon_{ijk} A_{(1)}^j \wedge A_{(1)}^k$.

Since the theory allows us to truncate to $U(1)^2$, we can include in our theory a vector-tensor multiplet with the Lagrangian given by⁴

$$\hat{e}^{-1} \mathcal{L}_6 = \hat{R} - \frac{1}{2} (\partial\phi_1)^2 - \frac{1}{2} (\partial\phi_2)^2 - \hat{V} + \frac{1}{4} \sum_{i=1}^2 X_i^{-2} (\hat{F}_{(2)}^i)^2, \tag{4}$$

where $X_i = e^{\frac{1}{2} \vec{a}_i \cdot \vec{\phi}}$ with $\vec{a}_1 = (\sqrt{2}, \frac{1}{\sqrt{2}})$ and $\vec{a}_2 = (-\sqrt{2}, \frac{1}{\sqrt{2}})$. The scalar potential is given by

$$\hat{V} = -\frac{4}{9} g^2 (X_0^2 - 9X_1 X_2 - 6X_0 X_1 - 6X_0 X_2), \tag{5}$$

where $X_0 = (X_1 X_2)^{-3/2}$. We consider the ansatz

$$\begin{aligned}
ds_6^2 &= -d\tau^2 + a^2 dx_j^2 + b^2 d\Omega_2^2, \\
F_{(2)}^i &= \lambda_i \Omega_{(2)},
\end{aligned} \tag{6}$$

where the functions a and b depend only on the co-moving time coordinate τ and $d\Omega_2^2$ is the metric for the unit 2-sphere S^2 . A cosmological solution can be obtained from the following first-order equations [4]

$$\begin{aligned}
\dot{\vec{\phi}} &= \sqrt{2} \left(-\frac{1}{2\sqrt{2}g} (q_1 \vec{a}_1 X_1^{-1} + q_2 \vec{a}_2 X_2^{-1}) b^{-2} + \frac{dW}{d\vec{\phi}} \right), \\
\frac{\dot{b}}{b} &= -\frac{1}{4\sqrt{2}} \left(\frac{3}{\sqrt{2}g} (q_1 X_1^{-1} + q_2 X_2^{-1}) b^{-2} + W \right), \\
\frac{\dot{a}}{a} &= \frac{1}{4\sqrt{2}} \left(\frac{1}{\sqrt{2}g} (q_1 X_1^{-1} + q_2 X_2^{-1}) b^{-2} - W \right),
\end{aligned} \tag{7}$$

³De Sitter supergravities as hyperbolic reductions of type IIB* and M*-theories have recently been obtained in [10].

⁴Though in general not the case, the truncation is consistent for the present purpose of constructing cosmological solutions.

provided that the two $U(1)$ charges satisfy $g(\lambda_1 + \lambda_2) = 1$. The superpotential is given by $W = \frac{q}{\sqrt{2}}(\frac{4}{3}X_0 + 2X_1 + 2X_2)$. In particular, if $\lambda_1 = g^{-1}$ and $\lambda_2 = 0$, then we can consistently set $\phi_1 = 2\phi_2$ and (7) can be solved explicitly. After the coordinate transformation $d\tau = \frac{3}{2}e^{\frac{3}{\sqrt{8}}\phi_2}(gt)^{-1}dt$, the metric of the solution can be expressed as

$$ds_6^2 = \widetilde{H}^{-\frac{1}{4}} H^{\frac{1}{4}} \left[-\frac{9}{4} \widetilde{H} H^{-1} \frac{dt^2}{(gt)^2} + (gt)^2 (dx_i^2 + \widetilde{H} g^{-2} d\Omega_2^2) \right]. \quad (8)$$

where

$$e^{\sqrt{8}\phi_2} = \frac{\widetilde{H}}{H} = \frac{1 + \frac{3}{4} \frac{1}{(gt)^2}}{1 + \frac{9}{4} \frac{1}{(gt)^2}}. \quad (9)$$

The solution can be viewed as an intersection of a spatial domain wall wrapped on Ω_2 , characterized by the function H , and an S2-brane characterized by the function \widetilde{H} . This is a smooth cosmological solution in which the co-moving time runs from an infinite past, which is $dS_4 \times S^2$, to an infinite future, which is a dS_6 -type geometry with the boundary $R^3 \times S^2$.

For general values of q_i , the first-order equations cannot be solved analytically. However, it is straightforward to find the fixed-point solution of $dS_4 \times S^2$ where b and $\vec{\phi}$ are constants. Using a numerical method, we have verified that there are cosmological solutions which run from $dS_4 \times S^2$ in the infinite past and are of dS_6 -type in the future. The ratio of the Hubble constant in the infinite past and future can be straightforwardly calculated. In particular, for $-\lambda_1 \gg \lambda_2$, the ratio is given by

$$\frac{H_{\text{past}}}{H_{\text{future}}} = \frac{\sqrt{2}}{16} \left(\frac{3}{2} \sqrt{-3 \frac{\lambda_1}{\lambda_2}} \right)^{3/4}, \quad (10)$$

which can be arbitrarily large. It is surprising that we can get a somewhat realistic cosmological model from a first-order system, which implies supersymmetry from the point of view of * theories [4].

2 $D = 4$ Yang-Mills de Sitter gravity from M-theory

Four-dimensional de Sitter spacetime arises in standard eleven-dimensional supergravity as a warped product with a hyperbolic 7-space [11]. The latter can be expressed as a foliation of two 3-spheres, on which $SU(2)$ Yang-Mills fields can reside. This

enables us to use the following reduction ansatz for $D = 11$ supergravity [13]:

$$ds_{11}^2 = \Delta^{\frac{2}{3}} ds_4^2 + g^{-2} \Delta^{\frac{2}{3}} d\theta^2 + \frac{1}{4} g^{-2} \Delta^{-\frac{1}{3}} \left[c^2 \sum_i (h^i)^2 + s^2 \sum_i (\tilde{h}^i)^2 \right], \quad (11)$$

$$F_{(4)} = 2g \epsilon_{(4)} - \frac{1}{4} g^{-2} \left(s c d\theta \wedge h^i \wedge *F_{(2)}^i - s c d\theta \wedge \tilde{h}^i \wedge *F_{(2)}^i \right. \\ \left. - \frac{1}{4} c^2 \epsilon_{ijk} h^i \wedge h^j \wedge *F_{(2)}^k + \frac{1}{4} s^2 \epsilon_{ijk} \tilde{h}^i \wedge \tilde{h}^j \wedge *F_{(2)}^k \right), \quad (12)$$

where $c = \cosh \theta$, $s = \sinh \theta$, $\Delta = \cosh(2\theta)$, and $*$ denotes the four-dimensional Hodge dual. σ_i and $\tilde{\sigma}_i$ are $SU(2)$ left-invariant 1-forms and the vielbein h^i and $SU(2)$ Yang-Mills field strengths $F_{(2)}^i$ have been defined in the previous section. It is straightforward to verify that the Bianchi identity $dF_{(4)} = 0$ and the equation of motion $d\hat{*}F_{(4)} = \frac{1}{2} F_4 \wedge F_{(4)}$ are satisfied provided that the $SU(2)$ Yang-Mills fields $A_{(2)}^i$ satisfy the lower-dimensional equations of motion $D*F_{(2)}^i = 0$, where the covariant derivative D is defined by $DV^i = dV^i + g \epsilon_{ijk} A^j \wedge V^k$, for any vector V^i .

The evaluation of the $D = 11$ Einstein equations of motion is substantially more complicated, and we have not performed the calculation in full detail. However, we have verified that the equations of motion work for the $U(1)$ subsector of the $SU(2)$ gauge fields. Combining the result, the lower-dimensional equations of motion can be obtained from the Lagrangian

$$e^{-1} \mathcal{L}_4 = R - \frac{1}{4} (F_{(2)}^i)^2 - 8g^2. \quad (13)$$

Thus, we have obtained four-dimensional Einstein $SU(2)$ Yang-Mills de Sitter gauged gravity from $D = 11$ by consistent Kaluza-Klein reduction on a hyperbolic 7-space [13].

A regular cosmological solution of (13) is given by [12]

$$ds_4^2 = H^2 (-f^{-1} dt^2 + f dx^2 + t^2 d\Omega_2^2) \\ F_{(2)}^3 = \frac{2\ell}{(tH)^2} dt \wedge dx, \quad *F_{(2)}^3 = 2\ell \Omega_{(2)}, \\ H = 1 + \frac{\ell}{t}, \quad f = \frac{4}{3} g^2 t^2 H^4 - 1. \quad (14)$$

For $g^2 \ell^2 = \frac{3}{64}$, this is smooth cosmological evolution between $dS_2 \times S^2$ and a dS_4 -type geometry with the boundary of $S^2 \times S^1$. It is straightforward to lift the solution back to $D = 11$ and obtain a regular cosmological solution in M-theory. The corresponding

metric is given by

$$\begin{aligned}
ds_{11}^2 = & \Delta^{\frac{2}{3}} H^2 \left(-f^{-1} dt^2 + f dx^2 + t^2 d\Omega_2^2 \right) + g^{-2} \Delta^{\frac{2}{3}} d\theta^2 \\
& + \frac{1}{4} g^{-2} \Delta^{-\frac{1}{3}} \left[c^2 \left(d\omega_2^2 + \left(\sigma_3 - \frac{2g}{H} dx \right)^2 \right) \right. \\
& \left. + s^2 \left(d\tilde{\omega}_2^2 + \left(\tilde{\sigma}_3 - \frac{2g}{H} dx \right)^2 \right) \right].
\end{aligned} \tag{15}$$

3 Discussion

The crucial issue regarding both of the above cosmological solutions is their stability. Although the * theories are necessary from the point of view of time-like T-duality, they suffer from an instability due to the ghost-like nature of the supergravity fields. It is of interest, therefore, to further study whether the stability is protected by time-like T-duality or “supersymmetry.” Our second example might be more appealing since it is embedded within standard eleven-dimensional supergravity. The optimistic view is that the time scale of the instability is large enough to, nevertheless, validate the cosmological solutions.

Metastable de Sitter vacua have recently been constructed in type IIB theory [14]. All moduli are frozen due to fluxes as well as corrections to the superpotential from Euclidean D-brane instantons or gaugino condensation. However, these techniques can only yield inflationary models under restrictive conditions [15]. If such moduli-freezing techniques can be incorporated into our smooth inflationary models, then this might greatly enhance the cosmological landscape of M-theory [16]. In particular, we would have cosmologies which smoothly evolve (without topological transitions) between de Sitter-type spacetimes of different dimensionalities.

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References

- [1] C.M. Hull, *Timelike T-duality, de Sitter space, large N gauge theories and topological field theory*, JHEP **9807**, 021 (1998), hep-th/9806146.
- [2] C.M. Hull, *Duality and the signature of space-time*, JHEP **9811**, 017 (1998), hep-th/9807127.
- [3] S.J. Gates, Jr. and B. Zwiebach, *Gauged $N = 4$ supergravity theory with a new scalar potential*, Phys. Lett. **B123**, 200 (1983).
- [4] H. Lü and J.F. Vazquez-Poritz, *From de Sitter to de Sitter*, hep-th/0305250.
- [5] G.W. Gibbons, ‘Aspects of supergravity theories,’ Published in *GIFT seminar on supersymmetry, supergravity and related topics*, edited by F. del Aguila, J.A. de Azcarraga and L.E. Ibanez, World Scientific (1984).
- [6] J. Maldacena and C. Nunez, *Supergravity description of field theories on curved manifolds and a no go theorem*, Int. J. Mod. Phys. **A16**, 822 (2001), hep-th/0007018.
- [7] C.M. Hull and N.P. Warner, *Noncompact Gaugings From Higher Dimensions*, Class. Quant. Grav. **5**, 1517 (1988).
- [8] M. Cvetič, H. Lü and C.N. Pope, *Gauged six-dimensional supergravity from massive type IIA*, Phys. Rev. Lett. **83**, 5226 (1999), hep-th/9906221.
- [9] L.J. Romans, *Massive $N=2a$ Supergravity In Ten-Dimensions*, Phys. Lett. **B169**, 374 (1986).
- [10] J.T. Liu, W.A. Sabra and W.Y. Wen, *Consistent reductions of IIB*/M* theory and de Sitter supergravity*, JHEP **0401**, 007 (2004), hep-th/0304253.
- [11] G.W. Gibbons and C.M. Hull, *de Sitter space from warped supergravity solutions*, hep-th/0111072.
- [12] H. Lü, C.N. Pope and J.F. Vazquez-Poritz, *From AdS black holes to supersymmetric flux-branes*, hep-th/0307001.

- [13] H. Lü and J.F. Vazquez-Poritz, *Four-dimensional Einstein Yang-Mills de Sitter gravity from eleven dimensions*, hep-th/0308104.
- [14] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, Phys. Rev. **D68**, 046005 (2003), hep-th/0301240.
- [15] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S.P. Trivedi, *Towards inflation in string theory*, JCAP **0310**, 013 (2003), hep-th/0308055.
- [16] L. Susskind, *The anthropic landscape of string theory*, hep-th/0302219.